Unit Writing Guide

Instructional Unit Design Outline

Unit Title: PROBLEM SOLVING

Unit Designer(s): Brian Twitchell

Grade Level(s): 4 - 12 Time Span: 5+ Days

Content Area(s): In addition to the highlighted topic below, other content areas may be included depending on the choice of problems by the teacher for this unit.

Career Prep	Health and PE	M & C Languages	Social Studies
English Language Arts	Mathematics	Science & Technology	Visual & Performing Arts

Common Core State Standard(s) focus:
Domain: Standards of Mathematical Practice
Standard: MP1: Make sense of problems and persevere in solving them.
Content Standard: Dependent on the grade level and problems chosen for unit. See Appendix for sample.

Summary of Unit:

Describe the unit using verbs that align with the Standard(s) being assessed:
Students will learn the four-step problem solving process by George Polya. They will be able to read a problem with understanding, they will be able to create a plan to solve the problem, they will be able to carry out their plan, and finally be able to check their solution, and if needed, go back to attempt a solution as often as needed until they have solved the problem.

Instruction will be based on a total of 6 problems labeled Problem 0, Problem 1, Problem 2, and so forth through Problem 5. Problem 0 is for daily use as class wide discussion as described below. **Problems 1 - 5 are for days 1 - 5 for smaller group work. Ultimately, each problem will be fully solved by the groups, but the focus each day is on a different portion the problem solving strategy. Each day a new problem is started, and the response up thro the current step will be submitted as an exit ticket. For example, on Day 2, students will submit work showing an "Understanding" of problem 2, as well as "Making a Plan" for problem 2.**

How does the content relate to real-life experiences?
Problem solving is one of the most important, yet least liked, aspects of mathematics. Problem solving occurs in all aspects of life from things as simple as "how much flour do I need for a double batch of cookies," to problems related to societal issues such as causes of climate change and addressing poverty both in our country and around the world. The skill of problem solving is one that is critical for our students to have.

How do the math concepts connect to concepts which are prior knowledge?
This unit is presented as if students have had no experience with the four-step problem solving method. The initial design does not contain specific grade level problems, but can be adapted to a variety of grade levels. This makes the plan adaptable to multiple grade levels.
A sample of problems for a 6th grade classroom is included in Appendix A.

Format Learning targets:

- I can use strategies to read new problems and understand what is being asked.
- I can make an estimate where appropriate for solving a problem.
- I can use various strategies to make a plan for solving a problem.
- I can carry out my plan.
- I can check my answer to see if it makes sense.
- I can make sure I have answered the question.
- I can revise my plan and attempt another solution if my first plan and solution did not work.
- I can keep working until I successfully solve the problem.

Rubrics for the standards
A suitable rubric for both formative and summative assessments is included at the end of this document, prior to Appendix A.

Declarative and Procedural Knowledge that students will acquire:

Declarative Knowledge	**Procedural Knowledge**
The four step problem solving process Understand Make a plan Carry out the plan Check your solution From the Standard MP1 Make Sense Persevere Solve	Make Sense: Students will have to demonstrate a clear understanding of the problem they are presented with. Persevere: Students will have to demonstrate the ability to make multiple attempts at solving a problem, seeking assistance from other peers as needed, until they have solved the problem.

Key Pre-Requisites	
Declarative Knowledge	Procedural Knowledge
These prerequisites will be dependent on the problems chosen. See Appendix A for a 6th grade sample.	

How will students provide evidence of their developing understandings?
(Formative Assessment)

The first four days will focus on one of the steps of the four-step method. Students will work in groups to complete the various tasks throughout the week. The lessons will be designed to help students better understand the meaning of the step being learned. Each day formative assessment will occur in the following ways:

- The teacher will listen and observe student interactions and discussions.
- Students will engage in class wide discussions.
- A set of four problems will be utilized throughout the week. On Day 1, the focus will be on understanding Problem 1, but no additional work will be done at this time. On Day 2, students will apply the first step to Problem 2, and then focus on making a plan for problem 2. On Day 3, students will apply the first two steps to Problem 3, and then focus on solving Problem 3. On Day 4, students will apply the first 3 steps to problem 4 and then focus on checking their solution to problem 4.
- Once problem 4 is completed, students will go back to work on the remaining parts of problems 1 - 3.
- A final formative assessment will be given at the end of the unit on Day 5 with a new problem students have not seen. They will be given multiple days to solve this problem.

Teaching and learning experiences used to help students understand:
(List all activities)

Day 1:

There will be a common problem (Problem 0) used over the first four days for the teacher and class to focus on the four steps. On the first day, the teacher will present the problem, and lead the class in a discussion on understanding the problem. Questions to ask include things such as:

- Are there words or ideas we do not understand or we wonder about?
- Can we highlight the important words?
- Is there information that we do not need?
- Do we know what the problem is asking us to find?
- Can anyone make an estimate to the answer?

Once the class has addressed the understanding portion of Problem 0, the class will be divided into groups and each group given Problem 1. As a group, they are to discuss the understanding process for this problem. A template will be given to each student where they can record their answers. (See final two pages of Appendix A). This becomes the exit ticket for the lesson.

Day 2:

The teacher will have reviewed the exit tickets from Problem 1 and can start by addressing anything that turns up. The teacher will then lead the class in a discussion on making a plan to solve problems. The class should generate a list of possible strategies that they might use on any problem. Some example strategies are:

- Use a number line
- Draw a picture
- Make a table or list
- Solve with an easier problem
- Work backwards
- Guess, check, and revise
- Model with manipulatives
- Look for a pattern
- Write an equation

Once this class has generated a list of strategies, the teacher should then review Problem 0 and lead the class in a discussion on what strategies seem appropriate as well as writing out an actual plan they might use. The teacher will NOT have them work on solving Problem 0 at this time, and the teacher should NOT guide the students to a plan that the teacher knows will work. If they generate a plan that will NOT lead to a solution, that is even better. Once this plan is completed, students will again be broken into groups and given Problem 2. They should first go through the understanding portion of the process, and then work on making a plan. Once their plan is complete, it should be turned in as an exit ticket.

Day 3:

Following the pattern of Day 2, the teacher should address anything from the exit tickets that turned up - again, not to correct plans that won't work, but to be sure plans are complete and clear. The teacher will then talk with the class about solving a problem, and emphasize that sometimes our solutions do not work. If that happens here, then we go back to step 2, make a new plan, and try another solution. Problem 0 should be reviewed, and the understanding portion and the plan the class created should be revisited. As a class, they can work on solving this problem. Again, if the class comes to an incorrect solution, do not guide them to the correct solution, Rather, allow the "Check your answer" portion of the problem solving process on Day 4 to address this issue. If the class comes to a point where they CANNOT solve the problem, then they need to go back to step 2 and create a new plan.
Once the class has a solution, again break into groups and provide each group with Problem 3. Students should work through steps 1, 2, and 3 and turn in their work as an exit ticket.

Day 4:

Follow the same pattern as earlier. Checking the answer should have several components. See how many of the following list the class can generate:

- Does the answer make sense?
- Did we double check the work?
- Is the solution clear to someone reading the paper?
- Are units included if appropriate?
- Does the solution connect to the plan?
- Did we answer the question?

Have the class apply these ideas to Problem 0. If necessary, have them go back to step 2 and create a new plan and solve again. When the problem is finally completed, break them into groups one more time and give them problem 4 and have them as a group go through the entire 4 step process.

Day 5:

During this class, students should revisit problems 1 - 3 that were started earlier in the week. They should generate complete solutions to these problems. When they are done, they should be given Problem 5 to work on. There should NOT be an expectation that this problem will be completed in a single class period, but encourage them to use multiple days if needed to solve the problem. Provide 15 - 30 minutes each day out of the regular class period for students to work on their problems. Groups can talk to other groups as needed.

Day 6 and Beyond:

The website www.cuethink.com is a problem solving website that follows this process. It embeds all four steps, but also allows for students to make an audio recording as they solve their problem, so they are explaining in words their solution. These "thinklets," as CueThink calls them, are then visible to the rest of the class to be able to view, comment on, and learn from. Students can also go back and re-do problems if mistakes are pointed out by others, or if they learn something from watching other students thinklet.

Provisions for extending learning:
Explain how these learning activities exceed the expectations.

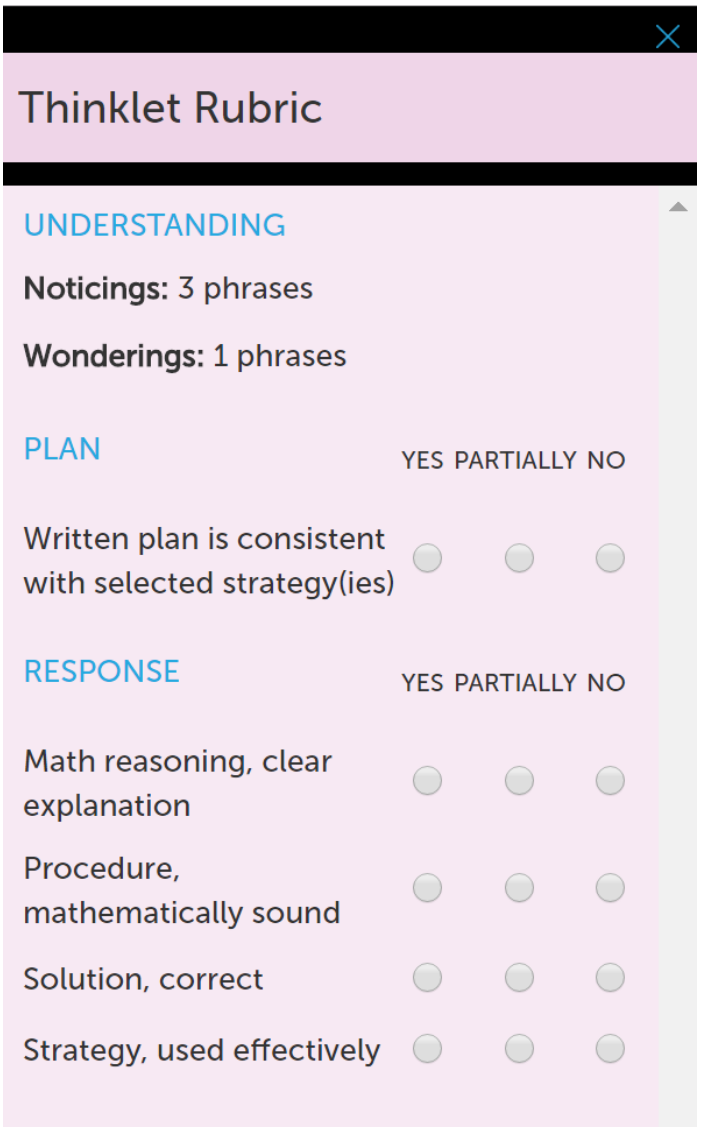
Students can be encouraged to see how they can apply their solution for the given problem to other problems. Some problems can be found that have extensions built in. See problems 1, 2, and 4 in the Appendix for some problems with extensions built in.
Some other possible ideas are:

- Generalize the solution to other similar problems
- Use their work to create additional problems
- Make a conjecture
- Apply the solution to a real-world problem

How will technology be used to increase student achievement?
•What tools are used? •How are the tools used?

Students in Grades 4 - 12 in the district will have one-to-one Chromebooks issued. These are touch devices, so students can either type or digitally write as desired.

The website www.cuethink.com will be a powerful technology to implement following this unit. In addition to allowing students to comment on each other's work, it also allows teachers to comment and score the student work. The built in rubric of Cuethink allows a quick scoring of the plan and the response as shown below. Note that this rubric is fundamentally different from the rubric provided at the end of this document, in that it is broken down into different skills surrounding primarily the solution portion of the process. While it provides good feedback to students, the teacher should consider if this is sufficient for their needs for this unit.



Instructional resources
This is a rich and full listing of actual resources for the unit.

Chromebooks
Word Wall for key terms of unit
Laptops for CueThink Activity
Chart Paper/Whiteboard/Digital Projector for showing problems and posting student discussion
www.cuethink.com - Problem Solving website - subscription required
http://map.mathshell.org/stds.php - a resource of many problems filtered by grade/content, as well as the Standards of Mathematical Practice.
http://illuminations.nctm.org/Default.aspx - Problems searchable either by NCTM standards or the CCSS
https://nzmaths.co.nz/problem-solving - Problems at various levels from New Zealand
http://cemc.uwaterloo.ca/resources/potw.php - University of Waterloo, Canada, Problem of the Week site

Interventions
Identify alternative teaching/learning opportunities for struggling students and gifted/advanced students.

Problems can be adapted for both struggling students and gifted students. Although it is possible to have different leveled problems for the problems 1 - 4, it is recommended to keep all problems the same for the instructional piece. This would allow for a deeper class discussion on follow up. The teacher can choose problems at a lower level than the class is at, especially if there are numerous students performing at a lower level. The final formative assessment problem could be done with different problems for different groups, but again this limits the ability of a class wide discussion.

Students who still struggle with showing proficiency on this mathematical practice could be given additional attempts to show proficiency with different level problems later in the year. In fact, all students should continue to be assessed on this practice throughout the year in a variety of different ways.

When students transition to CueThink the problem could be constructed with a problem for all students, and optional extensions for students who finish the base problem. This would allow all students to solve the problem, but also allow for more challenging options for those students who wanted the challenge. Problems 1, 2, and 4 in the Appendix include extensions that can be used for this purpose.

RUBRIC FOR BOTH FORMATIVE AND SUMMATIVE ASSESSMENTS:

<table>
<tr><td>Score 4.0</td><td colspan="2">In addition to Score 3.0, in-depth inferences and applications that go beyond what was taught.
● Student is able to solve an extension to this problem, if provided OR
● Student is able to create a generalization of the problem OR
● Student is able to create a more in depth problem based on the original problem OR
● Student is able to apply this problem to a meaningful real-world situation</td></tr>
<tr><td></td><td>3.5</td><td>In addition to score 3.0 performance, in-depth inferences and applications with partial success.</td></tr>
<tr><td>Score 3.0</td><td colspan="2">CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them.
● Student correctly solves the problem, revising the plan and making multiple attempts as necessary.
● Student checks the answer to be sure it is reasonable and correctly explained.
The student exhibits no major errors or omissions.</td></tr>
<tr><td></td><td>2.5</td><td>No major errors or omissions regarding 2.0 content and partial knowledge of the 3.0 content.</td></tr>
<tr><td>Score 2.0</td><td colspan="2">There are no major errors or omissions regarding the simpler details and processes as the student:
● Student creates a plan for solving the problem
However, the student exhibits major errors or omissions regarding the more complex ideas and processes.</td></tr>
<tr><td></td><td>1.5</td><td>Partial knowledge of the 2.0 content, but major errors or omissions regarding the 3.0 content.</td></tr>
<tr><td>Score 1.0</td><td colspan="2">With help, a partial understanding of some of the simpler details and processes and some of the more complex ideas and processes.
● Student is able to demonstrate they understand the problem.</td></tr>
<tr><td></td><td>0.5</td><td>With help, a partial understanding of the 2.0 content, but not the 3.0 content.</td></tr>
<tr><td>Score 0.0</td><td colspan="2">Even with help, no understanding or skill demonstrated.</td></tr>
</table>

Appendix

Below are sample problems for a Grade 6 unit. The problems are given, as are links to full resources where available.

Problem 0 is for class wide discussion.

Problems 1 - 5 are for days 1 - 5 as described above. Ultimately, each problem will be fully solved by the groups, but the focus each day is on a different portion of the problem solving strategy. Each day a new problem is started, and the response up through the current step will be submitted as an exit ticket. For example, on Day 2, students will submit work showing an "Understanding" of problem 2, as well as "Making a Plan" for problem 2.

Problem 0

Content Standard(s)

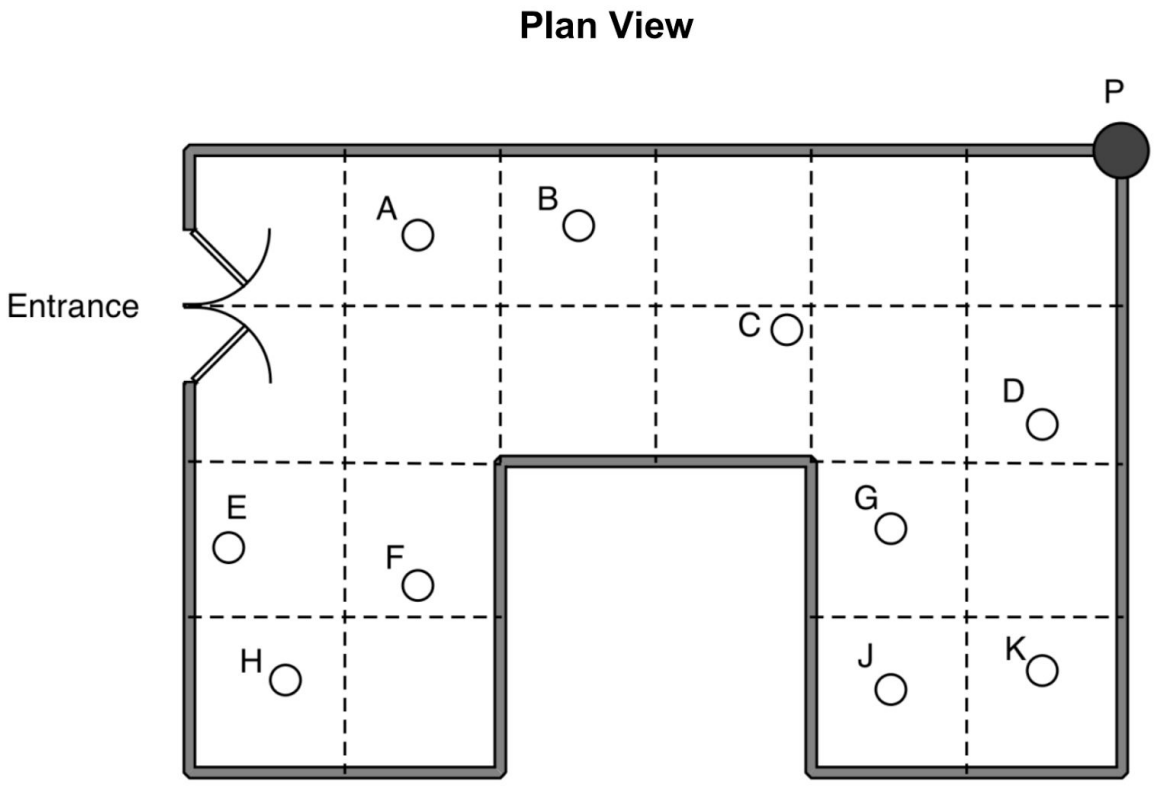
6.G: Solve real-world and mathematical problems involving area, surface area, and volume.
6.RP: Understand ratio concepts and use ratio reasoning to solve problems.

Key Pre-Requisites

Declarative	Procedural
Area Line of Sight Percent	Compute areas of rectangles and triangles Compute percent based on areas Understand line of sight for camera

Problem

Security Cameras A shop owner wants to prevent shoplifting. He decides to install a securi camera on the ceiling of his shop. The camera can turn right round through 360 in all directions. The shop owner places the camera at point P, in the corner of the shop. The pla view below shows where ten people are standing in the shop.



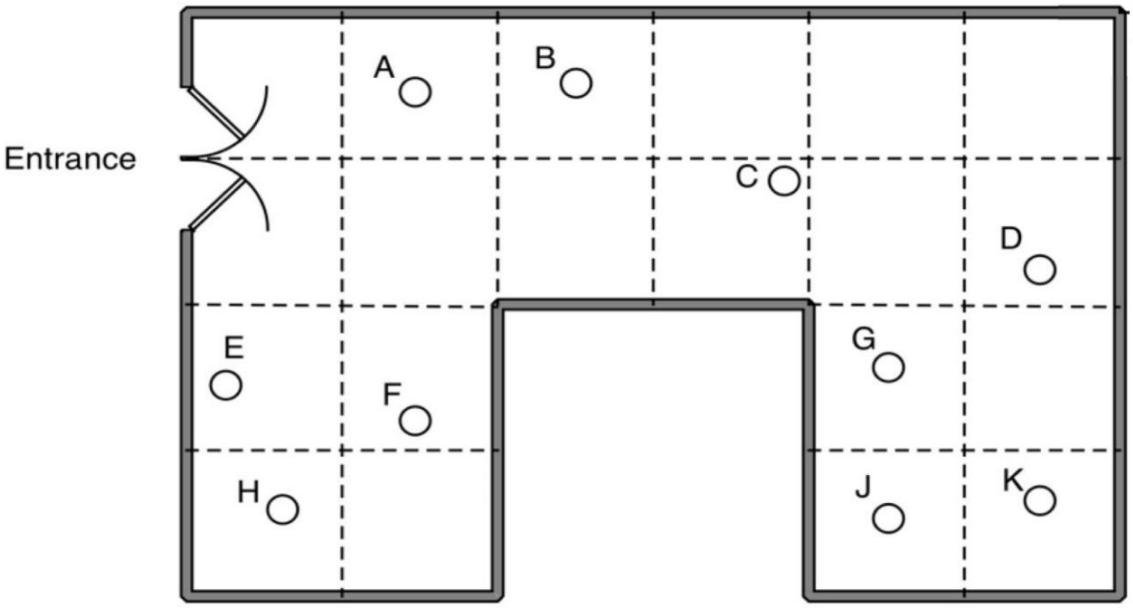
1. Which people in the shop cannot be seen by the camera at P? Explain your answer, show

clearly on the diagram how you know.

2. The shop owner says, "15% of the shop is hidden from the camera." Show clearly that h correct.

3. The shop owner decides to move the camera so that it can see as much of the shop as possible.

a. On the plan view below show the best place for the camera so that it can see as much of t shop as possible.



b. Explain how you know that this is the best place for the camera. What percentage of the shop cannot be seen with the camera in this new position?

Link to problem and other resources

http://map.mathshell.org/download.php?fileid=1584

Problem 1

Content Standard(s)

5.OA.B: Analyze patterns and relationships.

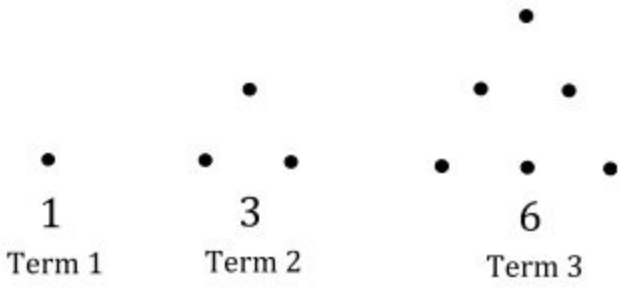
Key Pre-Requisites

Declarative	Procedural
Sequence **odd** **even**	**Understand extending patterns**

Problem

The numbers 1, 3, 6, 10, 15, 21, and so on, form a sequence. That is, there is a rule which te you how to find each number, or term in the sequence, from the previous term.

a) These numbers are sometimes called Triangular Numbers because they can be represen by dots arranged in triangles. Here are the first three terms:



Draw the dot-triangles that would represent the next three terms, 10, 15, and 21.

b) Write a pattern rule for this sequence.

c) Enter the next five numbers in the sequence in the table.

Term	Number
1	1
2	3
3	6
4	10
5	15
6	21
7	
8	
9	
10	
11	

Extension: What is the pattern of odd and even numbers in this sequence? Why does this happen?

Link to problem and other resources

http://cemc.uwaterloo.ca/resources/potw-strands/2016-17/English/POTWB-16-Combined5.pdf page 114

Problem 2

Content Standard(s)

6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressio
5.OA.B: Analyze patterns and relationships.

Key Pre-Requisites

Declarative	Procedural
add subtract money value	guess and check organizing information

Problem

To encourage Dennis to work harder at his math problems, his mother promised she would him 10 cents for each right answer, but subtract 5 cents for each wrong answer.

a) If he earned 20 cents after doing 32 problems, how many problems did Dennis get corre How many did he get wrong?

b) Dennis answered another 32 problems. What is the least number of problems he would l to get correct in order to earn more than one dollar?

Extension: Think about how to explain this mathematically in an efficient way.

Link to problem and other resources

http://cemc.uwaterloo.ca/resources/potw-strands/2016-17/English/POTWB-16-Combined5 df - Page 120

Problem 3

Content Standard(s)

6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressio

Key Pre-Requisites

Declarative	Procedural
	Organizing information

Problem

Sports teams often celebrate a good play or a winning goal by giving one another "high five

a) Emelia's basketball team won their game. Everyone gave everyone else on the team (5 players in total) a single high five. How many high fives were exchanged?

b) If the "spare" player on Emelia's team were included in the high fives, how many exchan would occur?

c) Yousef scored the winning goal for his soccer team. If all 11 players gave high fives to ea other, how many high fives were exchanged?

Link to problem and other resources

http://cemc.uwaterloo.ca/resources/potw-strands/2015-16/POTWB-15-Combined5-6.pdf pa 23

Problem 4

Content Standard(s)

5.MD.A: Convert like measurement units within a given measurement system.
6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.

Key Pre-Requisites

Declarative	Procedural
Seconds in a minute Minutes in an hour Hours in a day Days in a year	Understand conversions between time units

Problem

A good way to ensure that you do a complete job of washing your hands is to sing the `Hap Birthday' song while you wash.

a) For approximately how long will you wash your hands each time?

b) If you wash your hands 6 times per day, how many seconds will you spend washing in to each day?

c) Supposing you could reach the bathroom sink to wash your own hands by your 4 th birthday, and that you have done so ever since as in parts a) and b), how many hours and minutes have you spent washing your hands so far?

Extension: If you live to be 75 years old, how many days of handwashing will you have do by then?

Link to problem and other resources

http://cemc.uwaterloo.ca/resources/potw-strands/2015-16/POTWB-15-Combined5-6.pdf pa 41

Problem 5

Content Standard(s)

6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.

Key Pre-Requisites

Declarative	Procedural
Square Minutes Dimensions Rectangular	Finding areas of squares Finding areas of rectangles Finding factors of 900

Problem

Moe helps out the seniors Larry and Curly on his block by mowing their lawns during the summer. He can mow a square lawn of side length 30 m in 45 minutes.

a) If he works at the same rate, how many minutes will it take Moe to mow a square lawn o side length 60 m?

b) What are the dimensions of possible rectangular lawns with the same area as the lawn o side 30 m, and dimensions which are whole numbers? Which of these are you likely to see i the city?

Link to problem and other resources

http://cemc.uwaterloo.ca/resources/potw-strands/2015-16/POTWB-15-Combined5-6.pdf pa 37

Exit Ticket **NAME: _____**

Problem: _____ **Date: _____**

Directions: Complete each box based on the four-step problem solving rubric we worked on in class. Use additional paper as needed.

Understand:

Check here if additional paper is used: ☐

Plan:

Check here if additional paper is used: ☐

Solve:

Check here if additional paper is used: ☐

Check:

Check here if additional paper is used: ☐